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Creating clear designs: a graph-based algorithm and a catalogue of clear compromise plans

Zur Konstruktion unvermengter Versuchspläne: ein graph-basierter Algorithmus und ein Katalog unvermengter Kompromiss-Designs (englischsprachig)

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Creating clear designs: a graph-based algorithm and a catalogue of clear compromise plans

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Abstract

A graph-based algorithm is proposed for designing experiments such that a pre-specified set of 2-factor interactions is clear of aliasing with any main effects or two-factor interactions (clear design). The “clear interaction graphs” used in the algorithm are unique for each design and different in nature from the well-known Taguchi linear graphs. Based on published catalogues of 2-level fractional factorials, enhanced by these graphs, a search algorithm finds an appropriate clear design or declares its non-existence. The approach is applied to creation of a catalogue of minimum aberration clear compromise designs, which is also of interest in its own right.

Key words: clear 2-factor interactions, compromise plan, design of experiment, linear graph, clear interaction graph

1. Introduction

In designed industrial experiments, 2-level fractional factorial plans play an important role, since they are both parsimonious in the number of runs and – if designed well – allow estimation of effects of interest. In regular 2-level fractional factorial designs, a small number of runs is achieved by starting from a full factorial in a few, say $m–k$, factors and assigning k additional factors to interaction columns in this design. This generates perfect confounding: The 2^m effects of the full model with a constant, the main effects and all interactions up to order m can only be estimated as $2^{m–k}$ sums of 2^k effects each, i.e. each effect is perfectly confounded with $2^k–1$ other effects. It is often true that interaction effects of more than two factors can be neglected, so that it is not considered problematic if an interaction among three or more factors is confounded with an effect of interest. Thus, it is customary to call effects “clear”, if they are not confounded with main effects or 2-factor interactions (2fis) (cf. e.g. Wu and Chen 1992).

There are many ways of assigning m factors to $2^{m–k}$ runs. The k factors to be added to the full factorial in $m–k$ factors generate $2^k–1$ words (=groups of factors whose interaction is perfectly aliased with the overall mean). The length of the shortest word is called the resolution of the design and is denoted as a roman numeral. Resolution V designs do not confound any main effects or 2fis with each other, in resolution IV designs, 2fis can be confounded with each other, and resolution III designs even confound main effects with 2fis.

In this article, like in much of the related literature, it is assumed that interactions of order higher than two are negligible. Under this assumption, resolution V designs are generally considered adequate, if 2fis are to be estimated. However, they are often not affordable (16 runs for 5 factors, 32 runs for 6 factors, 64 runs for 7 or 8 factors, 128 runs for 9 to 11 factors, 256 runs for 12 to 17 factors). As an aside, note that there are non-regular fractional factorial plans that allow orthogonal estimation of all main effects and two-factor interactions for up to 15 factors in 128 runs or up to 19 factors in 256 runs (cf. e.g. Mee 2009, Chapter 8.2). These are not covered here.

Various authors (e.g. Addelman 1962, Wu and Chen 1992, Ke and Tang 2003, Wu and Wu 2002, Ke, Tang and Wu 2005) have discussed the possibility of devising resolution IV designs such that a pre-specified set of 2fis – called the requirement set in the sequel – can be estimated. While it is often assumed that some 2fis are negligible, this article will make no such assumption. Instead, the requirement set will always contain all main effects and those 2fis which are *of special interest*, without any assumption regarding which 2fis are active. For example, in a robustness experiment with the purpose to find settings of so-called control factors such that the so-called noise factors have as little impact as possible, interactions between control and noise factors may be of special interest, i.e. the requirement set might consist of all main effects and these 2fis, without necessarily assuming negligibility of other 2fis. All effects from the requirement set are estimable, if they are "clear", i.e. if they are not confounded with any main effect or 2fi, and a design that keeps a requirement set clear is called a clear design in the sequel. Clear designs can be of resolution IV, because 2fis from outside the requirement set need not be clear. This article provides a new type of graphs, clear interaction graphs, and an algorithm that uses these for finding clear designs.

Section 2 gives a concise overview of 2-level fractional factorial designs, their quality criteria and estimability requirements, and introduces clear compromise plans. In Section 3, Taguchi (1988) linear graphs are briefly sketched, and the new "clear interaction graphs" (CIGs) are introduced. Section 4 proposes a simple and usually fast algorithm for finding clear designs based on design catalogues enriched by the CIGs and a subgraph isomorphism search algorithm, while Section 5 presents a catalogue of minimum aberration clear compromise designs that has been derived using the algorithm of Section 4. Section 6 concludes the article with final remarks.

2. Two-level fractional factorials

2.1. Regular 2-level fractional factorial designs

The starting point for a regular 2-level fractional factorial design for m factors is a full factorial in 2^{m-k} runs for m-k 2-level factors, the levels for which are denoted as "-1" and "+1". The model matrix of the saturated model for the full factorial is usually denoted in the so-called "Yates order", which is the obvious continuation of the order **1 2 12 3 13 23 123** …, if **1**, **2**, **3** … are the m-k base factors of the full factorial. This matrix is called the "Yates matrix" in the following. Its columns consist of "-1" and "+1" entries, such that columns for interaction effects are obtained as products of the respective main effect columns, with base factor **1** a sequence of 2^{m-k-1} times the pairs -1,+1, base factor **2** a sequence of 2^{m-k-2} times the quadruples -1,-1,+1,+1, base factor 3 a sequence of 2^{m-k-3} times the octuple -1,-1,-1,-1,+1,+1,+1,+1, and so forth. Orthogonality of each pair of effects is easily verified by checking that their scalar product is 0.

Table 1: Generators corresponding to Yates matrix column numbers

	1 A	2 B	3 AB	4 C	5 AC	6 BC	7 ABC
8 D	9 AD	10 BD	11 ABD	12 CD	13 ACD	14 BCD	15 ABCD
16 E	17 AE	18 BE	19 ABE	20 CE	21 ACE	22 BCE	23 ABCE
24 DE	25 ADE	26 BDE	27 ABDE	28 CDE	29 ACDE	30 BCDE	31 ABCDE
32 F	33 AF	34 BF	35 ABF	36 CF	37 ACF	38 BCF	39 ABCF
40 DF	41 ADF	42 BDF	43 ABDF	44 CDF	45 ACDF	46 BCDF	47 ABCDF
48 EF	49 AEF	50 BEF	51 ABEF	52 CEF	53 ACEF	54 BCEF	55 ABCEF
56 DEF	57 ADEF	58 BDEF	59 ABDEF	60 CDEF	61 ACDEF	62 BCDEF	63 ABCDEF
64 G	65 AG	66 BG	67 ABG	68 CG	69 ACG	70 BCG	71 ABCG
72 DG	73 ADG	74 BDG	75 ABDG	76 CDG	77 ACDG	78 BCDG	79 ABCDG
80 EG	81 AEG	82 BEG	83 ABEG	84 CEG	85 ACEG	86 BCEG	87 ABCEG
88 DEG	89 ADEG	90 BDEG	91 ABDEG	92 CDEG	93 ACDEG	94 BCDEG	95 ABCDEG
96 FG	97 AFG	98 BFG	99 ABFG	100 CFG	101 ACFG	102 BCFG	103 ABCFG
104 DFG	105 ADFG	106 BDFG	107 ABDFG	108 CDFG	109 ACDFG	110 BCDFG	111 ABCDFG
112 EFG	113 AEFG	114 BEFG	115 ABEFG	116 CEFG	117 ACEFG	118 BCEFG	119 ABCEFG
120 DEFG	121 ADEFG	122 BDEFG	123 ABDEFG	124 CDEFG	125 ACDEFG	126 BCDEFG	127 ABCDEFG

There are many different ways to assign *k* additional (generated) factors to the columns of a Yates matrix with 2^{m-k} rows. Substantial research has been conducted in order to list non-isomorphic regular fractional factorials (cf. e.g. Chen, Sun and Wu 1993, Xu 2009), where two designs are considered isomorphic, if they can be obtained from each other by switching rows or columns or levels within columns. The non-isomorphic regular fractional factorials for *m* factors in 2^{m-k} runs are usually denoted as *m*-*k*.idno with an index number “idno” denoting the different non-isomorphic versions, and lower “idno” expressing better performance on some overall quality criterion. The most important quality criterion is resolution, and designs of equal resolution are ordered w.r.t. the pertinence of the most severe form of aliasing, measured by the so-called word length pattern, which is the frequency table of word lengths. The relevant criterion is called minimum aberration (MA) and makes sure that the number of shortest words is minimal (and successively so for the next shortest words in case of ties). The design catalogues used in this article are ordered by the MA criterion and listed in terms of Yates matrix column numbers (cf. Table 2 in the appendix). For readers used to generator notation, Table 1 translates the Yates matrix column numbers from Table 2 to generators, in which the base factors are denoted by capital letters. Note that generators can also be directly inferred from the binary representation of the Yates matrix column numbers, by using the positions of “1”s from the right as indicators which factors interact; for example, the binary representation of 12 is 1100, i.e. the third and fourth position from the right imply the interaction CD.

Another overall quality criterion for resolution IV designs is MaxC2, i.e. maximization of the number of clear 2fis. For small designs, MA and MaxC2 often coincide (cf. also Wu and Wu 2002). For large designs, however, there are various situations for which MaxC2 designs are much worse than MA designs in terms of aberration (cf. e.g. Block and Mee 2005). It has been argued that the MA criterion is a good surrogate for model robustness criteria (e.g. Cheng, Steinberg and Sun 1999). The author agrees with this view. Non-MA MaxC2 designs sacrifice some of the model robustness by creating stronger aliasing among the remaining 2fis. Therefore, it is recommended to use MA as the general quality criterion, and to consider clear 2fis on an as-needed base only, i.e. to only require certain specific 2fis to be clear, if there is a particular interest in their estimation. This is exactly the purpose of the CIG-based algorithm presented here.

2.2. The role of the requirement set

If some effects are considered particularly interesting, i.e. make up the requirement set for an experiment, this can have different backgrounds. The experimenter may have substantial prior knowledge, which allows to assume all effects outside the requirement set to be negligible. Creation of experimental plans for such situations have been treated by many authors, e.g. Addelman (1962) or Wu and Chen (1992).

In the authors experience, requirement sets often express the focus of interest rather than an assumption of negligibility regarding all effects outside the requirement set. In such situations, it is not advisable to lightheartedly make any negligibility assumptions, apart from the usual negligibility of higher order effects, which may be justifiable. This leads to the need for a clear design, i.e. a design in which the effects from the requirement set are neither aliased with any main effects nor with any 2fi. A comparison of this “Clear” approach with the approach assuming negligibility of effects outside the requirement set – called the “Distinct” approach – is treated in some detail in Grömping (2010c).

If a clear design is appropriate for the research question at hand, apart from this clearness request, the experimental plan should be as model robust as possible. Following the reasoning of the previous section, the experimental plan should thus be MA among the possible clear designs. In other words, the goal is to find the design with highest resolution and smallest aberration that is able to clearly accommodate the requirement set.

2.3. Clear compromise plans

The m experimental factors are divided into the two groups G1 and G2 with m_1 and m_2 factors respectively, $m_1 + m_2 = m$. For example, in a robustness experiment, G1 might contain control factors and G2 noise factors or vice versa. Four classes of compromise plans have been defined, the requirement sets of which contain all 2fis in

Class 1: G1xG1,
Class 2: G1xG1 and G2xG2,
Class 3: G1xG1 and G1xG2 or
Class 4: G1xG2.

The first three classes were introduced by Addelman (1962), the fourth by Sun (1993). Addelman considered estimability of the specified effects under the assumption that all 2fis outside the requirement set are negligible. This approach is not pursued here. Instead, interest here is in clear compromise plans which have also been investigated by Ke, Tang and Wu (2005).

Ke et al. proved that there are no clear resolution IV compromise plans of class 2. For the other three classes, they provided lower bounds for the number of runs for a given m_1, as well as upper bounds for m_1 for a given number of runs. Furthermore, they provided a small catalogue of clear compromise plans in 32 and 64 runs for class 3 that can also be used for classes 1 and 4 and can in some cases be adapted to special needs by moving a factor from group G2 to group G1 or simply by omitting factors (their tables 1 and 2). They supplemented this catalogue with a few additional class 4 clear compromise plans (their table 4). Their catalogues do not make any claims w.r.t. quality criteria of the resulting clear compromise plans. Designs that can be obtained from their catalogue directly or by moving or deleting the last factor(s) of a group are shown in bold italics in the tables of MA clear compromise plans in the appendix. It can be seen that almost all their directly catalogued designs are MA, while designs obtained by moving or deleting columns can often be improved upon.

3. Clear interaction graphs

Before introducing the new clear interaction graphs, the well-known Taguchi linear graphs (cf. e.g. Taguchi 1988 for an extensive but incomplete listing) are briefly summarized: linear graphs indicate maximum estimable models for each design. Each main effect is a vertex of the graph; each edge represents a 2fi that is estimable, if all 2fis not in the graph as well as all higher order interactions are assumed negligible. There are several linear graphs for any particular design, corresponding to several differently structured requirement sets. Wu and Chen (1992) mentioned the possibility to show edges that represent clear 2fis by a special line type.

For clear designs, as negligibility assumptions for main effects or 2fis are not permitted. Thus, a more efficient tool can be used: this article proposes an alternative type of graph, the "clear interaction graph" (CIG). There is only one unique CIG for each regular 2-level fractional factorial design. Again, the factors themselves are the vertices in the graph. The edges are defined by the clear 2fis, i.e. two vertices are connected by an edge, if and only if the 2fi of the respective two factors is clear. Thus, any resolution V graph has all pairs of vertices connected by edges, while resolution III or IV graphs may or may not have edges. Note that the CIG itself does not reveal whether a design is resolution III or IV; usually, resolution III designs should not be considered in CIG applications. It will therefore be assumed throughout this article, that only resolution IV or higher designs are considered.

Figure 1 shows two examples of CIGs: the graph for the MA resolution IV design 9-4.1 in 32 runs and 9 factors indicates that all interactions of the 9^{th} factor with any other factor are clear, whereas all interactions of both the 9^{th} and the 5^{th} factor with each other and all other factors are clear in the second best design 9-4.2. This implies that design 9-4.2 is usable for a class 3 or class 4 clear compromise plan with m_1=2 or a class 1 clear compromise plan with m_1=3. For m_1=1 or 2, respectively, the better design 9-4.1 can be used.

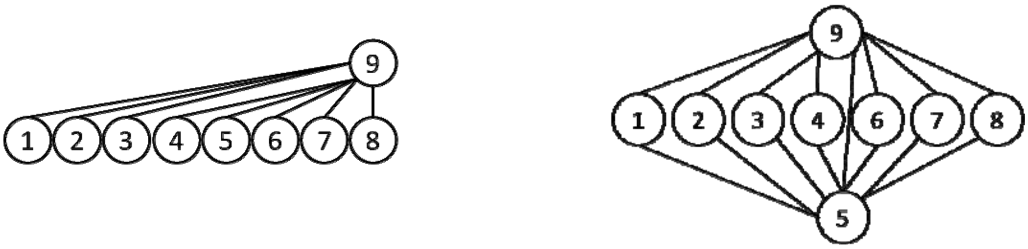

Figure 1: Clear interaction graphs for designs 9-4.1 (left) and 9-4.2 (right)
Vertices are labeled with factor numbers (cf. Table 2 for corresponding Yates matrix column numbers).

It is possible to provide CIGs both for an experiment's requirement set and for all catalogued designs. An intended experiment can be accommodated in a particular design, if its graph is contained in the designs graph, i.e. if there is a mapping of the requirement set graph vertices to the design graph vertices such that all edges in the requirement set graph are also present in the design graph. This comparison can be made by a subgraph isomorphism algorithm, which is the reason why the problem has been cast into graph form in the first place.

4. The proposed algorithm for finding clear designs

The algorithm requires that a complete catalogue of designs, ordered from best to worst, is available. In that case, the task of finding the best (resolution IV) design with the required 2fis clear can be solved by looping through the catalogued (resolution IV) designs from best to worst, and checking for each design whether the requirement set CIG is contained in the design's CIG.

Complete catalogues of designs ordered by the MA criterion exist in the literature (Chen, Sun and Wu 1993; Xu 2009 and the supplementary website). It would be possible to directly base a search algorithm on these published catalogues, calculating each design's CIG as part of the algorithm. However, including the CIGs into the catalogue leads to a substantial speed improvement. As CIGs are unique and can be represented e.g. by a two-row matrix with a column for each edge, the catalogues can easily be extended to include them. This has been done for designs with up to 128 runs for the implementation of the algorithm discussed below. The catalogues also contain the number of clear 2fis, which can be used to substantially reduce the search space. Obviously, once a complete catalogue of designs is available for the search process, the algorithm will either return the best possible design – after a successful identification of a subgraph mapping between the experiment graph and the design graph – or will exhaust the catalogue without finding a design which leads to the insight that there is no possibility to accommodate the requirement set within the catalogued designs. In other words, the algorithm is guaranteed to find the best existing design among the catalogued designs. The following two subsections illustrate application of the algorithm and discuss its implementation in a free open-source software by the author.

Note that the algorithm proposed here is very similar to a proposal made by Wu and Chen (1992) on the basis of linear graphs (i.e. making negligibility assumptions for effects outside the requirement set). As linear graphs are not unique and there can be many such graphs for any moderately-sized design, their proposal is much more resource-intensive than the algorithm proposed here.

4.1. A detailed application

The algorithm is now illustrated in detail, using an example by Ke and Tang (2003): *Example 1*: The 7 experimental factors A:temperature, B:moisture, C:pressure, D:thickness, E:time, F:size and G:speed are to be investigated, and the two interactions temperature*moisture and moisture*time are of special interest. This means that the requirement set consists of all main effects (i.e. a resolution IV design is needed) and the 2fis A*B and B*E. This requirement set defines the CIG in Figure 2.

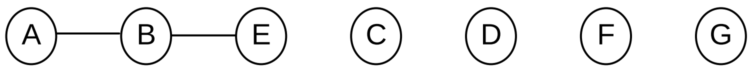

Figure 2: Requirement set CIG for Example 1

The task of assigning the experimental factors to the appropriate factor numbers of a suitable design now consists in finding the best possible design in the required number of runs for which the graph from Figure 2 is contained in the design's CIG.
The algorithm consists of the following two coarse steps:

1. Select the (next) best (criterion: MA) design that has at least as many clear 2fis as needed for the requirement set. If no (further) such design is found, declare impossibility of request.
2. Apply a subgraph isomorphism search algorithm (algorithm by Cordella et al. 2001 as implemented in Csardi and Nepusz 2006) in order to identify a mapping of experiment factors to design factors such that the requirement set is clear. If such a subgraph is found, the algorithm returns the solution design. Otherwise go to Step 1.

As a result from the algorithm, the experimenter gains a solution design or the definite answer that the request cannot be fulfilled (within the permitted search designs). In the latter case, the experimenter has to increase the number of runs.

Example 1, continued: The initial goal is to use a 16 run design.

> Step 1: There is no resolution IV design in 16 runs and 7 factors with at least two clear 2fis. The algorithm stops with the message that there is no solution.

The experimenter restarts the algorithm with run size increased from 16 to 32.

Step 1 again: The design 7-2.1 is selected as the best design with 2 clear 2fis.

Step 2: The design 7-2.1 has the CIG shown in Figure 3.

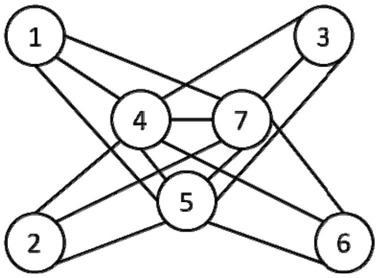

Figure 3: CIG for design 7-2.1

Comparison to the requirement set CIG yields many possible mappings of experiment factors to design columns, for example A=1, B=4, C=3, D=5, E=2, F=6, G=7 (as returned by the implementation of the algorithm described below). With this mapping, factors A to G are allocated to columns 1, 8, 4, 16, 2, 7, 27 of the 32 run Yates matrix (cf. Table 2).

If increasing the run size is not feasible, there are of course alternatives: Negligibility assumptions on effects outside the requirement set may be introduced, perhaps combined with a D-optimality approach, or non-regular orthogonal designs may yield a solution (e.g. for 12 to 15 factors in 128 runs or 18 and 19 factors in 256 runs, where non-regular resolution V designs exist, as mentioned in the introduction). In this example, if it appears adequate to assume negligibility of 2fis outside the requirement set, the requirement set can be accommodated in the 16 run MA design 7-3.1 using the default order of factors. However, this involves the risk of biased estimates, if the negligibility assumption is incorrect.

4.2. Implementation

The above-described algorithm has been implemented in **R**-package **FrF2** (Grömping 2007-2010; as part of the open source programming environment **R** (R development core team 2010). The algorithm uses **R**-package **igraph** (Csardi and Nepusz 2006, based on Cordella 2001) for the subgraph isomorphism checks within each iteration. As mentioned before, the published complete catalogues of non-isomorphic designs by Chen, Sun and Wu (1993, with personal communation by Don Sun regarding the resolution IV 64 run designs) and Xu (2009, with his supplement on the website for resolution IV 128 run designs up to 24 factors) have been enhanced by attaching its CIG and the number of clear 2fis to each design and serve as the basis for the search. Complete catalogues for up to 64 runs are part of the software itself. For 128 runs, the complete catalogue is too large to be included into the software; the software contains a few promising designs only (including all the ones needed for the MA clear compromise designs catalogued in this article), while the complete CIG-enhanced catalogue for up to 24 factors is distributed with the separate **R** package **FrF2.catlg128** (Grömping 2010b; the package links to the author’s website for the 24 factor catalogue).

5. Complete catalogue of smallest MA clear compromise designs

The CIG-based algorithm of the previous section has been used for creating a catalogue of clear compromise designs. Previously-published catalogues of compromise designs (Addelman 1962 for distinct designs, Ke et al. 2005 for clear designs) gave a small selection of designs for maximum values of m_1, together with instructions how to obtain further designs from these. Here, complete catalogues of clear compromise designs for designs with up to 24 factors in 128 runs are provided. These have been created using the algorithm from the previous section. Ke et al.'s (2005) bounds have been used for limiting the search for clear compromise plans to possible candidate designs.

The complete catalogue is presented in four tables in the appendix. Table 2 holds all base designs with their respective Yates matrix columns. Tables 3, 4 and 5 provide the complete listings of resolution IV clear compromise designs of the three classes for which such designs exist. These tables indicate, which factors of the respective base design belong to G1; Table 2 can be used for translating the column numbers from Tables 3 to 5 to Yates matrix columns.

5.1. Usage examples

Table 4 shows that the smallest MA 16 factor clear compromise plan of class 3 with m_1=2 can be obtained from the design 16-10.45 in 2^{16-10} = 64 runs using its columns 6 and 16. According to Table 2, these correspond to Yates matrix columns 32 and 60 for the G1 factors and Yates matrix columns 1 2 4 8 16 7 11 13 14 19 21 22 25 26 for G2. The design entry is set in bold italics, which indicates that an isomorphic design can also be obtained from Ke et al. (2005) by omitting the last G2 factor. The design has 77 words of length 4, which leads to quite heavy confounding (14 model matrix columns hold 6 2fis each, one holds 7 2fis). If a larger design can be afforded and is desired, the footnote to Table 4 indicates that the 128 run design 16-9.2 could be used with G1 columns 4 and 5, which corresponds to Yates matrix columns 8 and 16 for G1 and the remaining Yates matrix columns of the design for G2.

Let us now consider the analogous smallest MA class 1 clear compromise plan with 16 factors and m_1=3: according to Table 3 the MA design is based on the base design 16-10.8 and uses its columns 6, 13 and 16 for G1, which translates into Yates matrix columns 32, 25 and 63. This design has better aberration than the one obtainable from Ke et al. (2005). Nevertheless, it is still heavily confounded regarding some 2fis. If a 128 run design is desired for reducing the degree of confounding, the overall MA design 16-9.1 can be used, as it is listed further to the right in the table row for 16 factor designs: an MA clear class 1 compromise design in 128 runs for 16 factors with m_1=3 can be obtained from 16-9.1 using its columns 2, 3 and 5 for G1 (i.e. moving one column from G1 to G2 from the design with m_1=4). Within Tables 3 and 4, it is always permitted to move factors from G1 to G2, as was also stated by Ke et al. (2005) for their corresponding tables 1 and 2. Beware that this is not true for Table 5, where for example the design for 10 factors with m_1=4 can be based on the overall MA design

10-4.1, while m_1=3 requires using the design 10-4.3, which has worse aberration (cf. also the following section).

5.2. Observations regarding MA clear designs

The smallest MA designs in most cases require half the run size of a resolution V design. In some cases, designs with only a quarter of the runs of a resolution V design can also be used. For the latter cases, it may sometimes be desirable to double the run size for reducing the severity of confounding. In most such cases, the MA clear design can be obtained from the overall MA design in the doubled run size, e.g. from 16-9.1 in the class 1 example from the previous section. Often, this design is listed further to the right in Tables 3 and 4, in which case the MA column allocation for it can be obtained by moving one or more G1 column(s) to G2, as was the case for the class 1 example above. Where a larger MA design cannot be obtained in this way (some cases for Table 4, and all cases for Table 5), footnotes indicate how to allocate G1 factors, like in the class 3 example of the previous section.

For clear class 4 compromise plans, it is not permitted to move factors between G1 and G2, which is due to the absence of requirements regarding estimability of 2fis within G1. Consequently, there is no monotonicity in terms of m_1: for example, the overall MA design in 10 factors in 64 runs (10-4.1) can accommodate a clear class 4 compromise plan for m_1=2 or m_1=4, but not for m_1=3: The design has two non-overlapping words of length 4. Choosing all m_1=4 factors for G1 from the same 4-letter word, the design can be used as a clear compromise plan of class 4 for m_1=4, since all confounding is within G1 and within G2 only. When omitting one of these factors from G1, its interaction with the other three becomes important, and the design is not a clear class 4 compromise design for m_1=3. A G1 with m_1=2 factors can again be accommodated within this design, because there are two factors (positions 4 and 10) that do not occur in the two 4-letter words and thus have all their 2fis clear.

The designs that can also be obtained from Ke et al. (2005) have been set in bold italics in Tables 3 to 5. It can be seen that the larger MA designs catalogued here are in most cases better than those obtainable from Ke et al. The difference in aberration can sometimes be large, e.g. for a class 1 design with 12 factors and m_1=2, where the Ke et al. design obtained from their 17 run class 3 design by omitting the last 5 G2 columns would result in 18 words of length 4, as opposed to only six such words in the design catalogued here. For other cases, the difference is slight, for example for the class 3 design in 13 factors with m_1=2, where the design obtainable from the Ke et al. instructions has 26 words of length 4 as compared to 25 such words for the MA design.

Class 3 compromise plans are also class 4. As the class 3 requirement set is larger, the MA clear class 3 compromise plan is of course not necessarily MA for class 4. However, class 3 and class 4 MA clear compromise plans often coincide. Deviations occur in particular for relatively large values of m_1, because – as has been discussed above – there may be designs with confounded interactions within both G1 and G2 but clear interactions between groups. Class 3 compromise plans are also class 1. Alternatively, one can obtain a class 1 compromise plan with m_1 increased by one vs. a corresponding class 3 plan by moving one factor from G2 to G1. A comparison of

Tables 3 and 4 shows that MA clear class 1 compromise plans can often achieve better aberration than the corresponding class 3 plans.

6. Final remarks

Clear interaction graphs (CIGs) have been introduced, and an algorithm based on complete catalogues of CIGs has been proposed that is guaranteed to find the best existing clear design among the catalogued designs. Section 4.2 details an implementation of the algorithm for general use and indicates for which scenarios it has been implemented. The software is guaranteed to find the best existing resolution IV clear design with up to 64 runs or – if additional catalogues ordered by the MA criterion are available, the best existing clear design within those, for example with the additional complete catalogues for 128 run resolution IV designs with up to 24 factors provided in Grömping (2010b).

The **FrF2** implementation of the algorithm has been used for creating a complete catalogue of smallest MA clear compromise designs with up to 128 runs and 24 factors. This work serves as a demonstration of the usefulness of the CIGs and the CIG-based algorithm. On the other hand, the work on the catalogue has also been used to improve the software: those base designs that have shown up as yielding MA clear compromise designs should be generally useful for finding clear designs, even if no perfect compromise design is sought. The 128 run clear compromise plans from the catalogue in Tables 3 to 5 are based on 68 different base designs, 30 of which were not originally part of the selection of 128 run designs included in the software. They have been added to the software, which should also improve the chances for automatically finding better designs for general requirement sets. At the very least, the thus-enhanced software will automatically find all MA clear compromise designs catalogued in this article, without loading the additional complete catalogue of 128 run resolution IV designs.

Finally, apart from the CIGs and the proposed algorithm, the catalogue of clear compromise plans is of interest in its own right, as there are practical situations for which the experimental factors naturally fall into two groups with a certain pattern of 2fis being of interest. For example, in the robustness scenario mentioned in the introduction, control factors and noise factors are a natural choice for the two groups G1 and G2. The interactions between these two groups will be particularly interesting. Nevertheless, one may not be willing to assume negligibility of other interactions. In this case, a class 4 clear compromise design might be adequate. One may also want to estimate interactions among control factors, which will imply a class 3 clear compromise design, whenever one is not willing to assume negligibility of the 2fis among noise factors. Many other such situations are conceivable. Therefore, the catalogue of MA clear compromise designs can be quite useful for practitioners.

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Appendix: Tables of catalogues

Table 2: Resolution IV regular* base designs used in at least one clear compromise design (from the catalogues by Chen, Sun and Wu 1993 or Xu 2009, a personal communication by Don Sun (64 runs) and the web supplement to Xu 2009)

Design *m-k*.no.	Runs 2^{m-k}	Column numbers of factors 1 to *m* in Yates matrix
7-2.1	32	1 2 4 8 16 7 27
8-3.1	32	1 2 4 8 16 7 11 29
9-4.1	32	1 2 4 8 16 7 11 19 29
9-4.2	32	1 2 4 8 16 7 11 13 30
9-3.1	64	1 2 4 8 16 32 7 27 45
10-4.1	64	1 2 4 8 16 32 7 27 43 53
10-4.3	64	1 2 4 8 16 32 7 11 29 51
11-5.1	64	1 2 4 8 16 32 7 11 29 45 51
11-5.4	64	1 2 4 8 16 32 7 11 21 46 56
11-5.6	64	1 2 4 8 16 32 7 11 19 29 62
12-6.1	64	1 2 4 8 16 32 7 11 29 45 51 62
12-6.2	64	1 2 4 8 16 32 7 11 21 46 54 56
12-6.4	64	1 2 4 8 16 32 7 11 21 41 54 56
12-6.23	64	1 2 4 8 16 32 7 11 21 25 31 45
12-5.1	128	1 2 4 8 16 32 64 31 103 43 85 121
13-7.1	64	1 2 4 8 16 32 7 11 21 25 38 58 60
13-7.3	64	1 2 4 8 16 32 7 11 19 29 37 59 62
13-7.6	64	1 2 4 8 16 32 7 11 19 30 37 41 52
13-7.34	64	1 2 4 8 16 32 7 11 13 19 21 25 46
13-6.1	128	1 2 4 8 16 32 64 31 103 43 85 44 86
13-6.6	128	1 2 4 8 16 32 64 31 103 43 85 44 89
14-8.1	64	1 2 4 8 16 32 7 11 19 30 37 41 49 60
14-8.4	64	1 2 4 8 16 32 7 11 19 30 37 41 52 56
14-8.7	64	1 2 4 8 16 32 7 11 19 29 37 41 47 49
14-8.40	64	1 2 4 8 16 32 7 11 13 14 19 21 25 54
14-7.1	128	1 2 4 8 16 32 64 31 103 43 85 46 61 114
14-7.3	128	1 2 4 8 16 32 64 31 103 43 85 46 61 67
14-7.5	128	1 2 4 8 16 32 64 31 103 43 85 44 86 13
14-7.14	128	1 2 4 8 16 32 64 31 103 43 49 74 62 109
14-7.71	128	1 2 4 8 16 32 64 31 103 43 85 121 13 14
14-7.94	128	1 2 4 8 16 32 64 31 103 43 85 44 89 113
15-9.3	64	1 2 4 8 16 32 7 11 19 29 37 41 47 49 55
15-9.9	64	1 2 4 8 16 32 7 11 13 19 21 25 35 37 63
15-9.40	64	1 2 4 8 16 32 7 11 13 14 19 21 22 25 58
15-8.1	128	1 2 4 8 16 32 64 31 103 43 85 46 61 114 67
15-8.3	128	1 2 4 8 16 32 64 31 103 43 85 46 61 114 13
15-8.10	128	1 2 4 8 16 32 64 31 103 43 85 44 86 88 55
15-8.34	128	1 2 4 8 16 32 64 31 103 43 85 44 86 13 97
15-8.78	128	1 2 4 8 16 32 64 31 103 43 49 78 45 62 105
15-8.150	128	1 2 4 8 16 32 64 31 103 43 85 44 82 57 113
15-8.423	128	1 2 4 8 16 32 64 31 103 43 85 121 13 14 22
15-8.1221	128	1 2 4 8 16 32 64 31 103 43 85 44 89 113 125
16-10.2	64	1 2 4 8 16 32 7 11 19 29 37 41 47 49 55 59
16-10.8	64	1 2 4 8 16 32 7 11 13 14 19 21 25 35 37 63
16-10.45	64	1 2 4 8 16 32 7 11 13 14 19 21 22 25 26 60
16-9.1	128	1 2 4 8 16 32 64 31 103 43 85 44 86 88 53 110
16-9.2	128	1 2 4 8 16 32 64 31 103 43 85 46 61 114 67 78
16-9.80	128	1 2 4 8 16 32 64 31 103 43 85 44 86 88 55 56
16-9.890	128	1 2 4 8 16 32 64 31 103 43 85 44 82 57 113 89
16-9.1261	128	1 2 4 8 16 32 64 31 103 43 85 46 61 67 70 105
16-9.1413	128	1 2 4 8 16 32 64 31 103 43 85 46 56 88 79 55

Table 2, continued

Design *m-k*.no.	Runs 2^{m-k}	Column numbers of factors 1 to *m* in Yates matrix*
16-9.2913	128	1 2 4 8 16 32 64 31 103 43 85 121 13 14 22 19
16-9.5539	128	1 2 4 8 16 32 64 31 103 43 85 14 22 13 19 26
17-11.2	64	1 2 4 8 16 32 7 11 19 29 37 41 47 49 55 59 62
17-11.7	64	1 2 4 8 16 32 7 11 13 14 19 21 22 25 35 37 63
17-11.6 = 17-11.38**	64	1 2 4 8 16 32 7 11 13 14 19 21 22 25 26 28 63
17-10.1	128	1 2 4 8 16 32 64 31 103 43 85 46 61 114 67 78 116
17-10.1036	128	1 2 4 8 16 32 64 31 103 43 85 44 86 88 55 56 79
17-10.2407	128	1 2 4 8 16 32 64 31 103 43 85 44 82 57 113 89 105
17-10.5846	128	1 2 4 8 16 32 64 31 103 43 85 46 61 67 70 105 108
17-10.5924	128	1 2 4 8 16 32 64 31 103 43 85 46 56 88 79 55 104
17-10.9040	128	1 2 4 8 16 32 64 31 103 43 85 121 13 14 22 19 26
17-10.12633	128	1 2 4 8 16 32 64 31 103 43 85 14 22 13 19 26 28
18-11.1	128	1 2 4 8 16 32 64 31 103 43 85 46 61 114 67 78 116 121
18-11.23	128	1 2 4 8 16 32 64 31 103 43 85 44 86 88 53 38 79 98
18-11.95	128	1 2 4 8 16 32 64 31 103 43 85 46 61 114 67 78 116 105
18-11.5146	128	1 2 4 8 16 32 64 31 103 43 85 44 86 88 55 56 79 104
18-11.6381	128	1 2 4 8 16 32 64 31 103 43 85 44 82 57 113 89 105 123
18-11.14398	128	1 2 4 8 16 32 64 31 103 43 85 46 56 88 79 55 104 112
18-11.18050	128	1 2 4 8 16 32 64 31 103 43 85 121 13 14 22 19 26 28
19-12.1	128	1 2 4 8 16 32 64 31 103 43 85 46 61 114 67 78 55 58 86
19-12.2	128	1 2 4 8 16 32 64 31 103 43 85 46 61 114 67 78 55 58 97
19-12.10	128	1 2 4 8 16 32 64 31 103 43 85 44 82 54 56 88 78 123 125
19-12.488	128	1 2 4 8 16 32 64 31 103 43 85 44 86 25 105 58 106 54 114
19-12.9648	128	1 2 4 8 16 32 64 31 103 43 85 44 82 57 113 89 105 123 125
19-12.11319	128	1 2 4 8 16 32 64 31 103 43 81 45 49 118 73 94 56 104 127
19-12.12482	128	1 2 4 8 16 32 64 31 103 43 85 44 86 88 55 56 79 104 112
19-12.26381	128	1 2 4 8 16 32 64 31 103 43 45 87 46 88 55 56 79 104 112
20-13.1	128	1 2 4 8 16 32 64 31 103 43 85 46 61 114 67 78 55 58 86 91
20-13.2	128	1 2 4 8 16 32 64 31 103 43 85 46 61 114 67 78 55 58 97 108
20-13.11	128	1 2 4 8 16 32 64 31 103 43 85 44 82 54 56 88 78 123 125 104
20-13.43452	128	1 2 4 8 16 32 64 31 103 43 81 44 93 21 13 19 49 14 25 28
20-13.47458	128	1 2 4 8 16 32 64 31 103 43 45 87 46 88 55 56 79 104 112 127
21-14.1	128	1 2 4 8 16 32 64 31 103 43 85 44 82 54 56 88 78 123 125 104 25
21-14.4	128	1 2 4 8 16 32 64 31 103 43 85 44 86 88 53 78 58 83 97 28 104
21-14.8	128	1 2 4 8 16 32 64 31 103 43 85 44 82 54 56 88 78 123 125 104 113
21-14.68031	128	1 2 4 8 16 32 64 31 103 43 81 44 93 21 13 19 49 14 25 28 61
22-15.1	128	1 2 4 8 16 32 64 31 103 43 85 44 86 88 53 78 58 83 97 28 104 114
22-15.7	128	1 2 4 8 16 32 64 31 103 43 85 44 86 88 53 78 58 83 97 28 104 112
22-15.8509	128	1 2 4 8 16 32 64 31 103 43 49 74 124 7 61 84 13 67 82 37 62 94
22-15.118181	128	1 2 4 8 16 32 64 63 71 25 104 30 41 78 112 15 49 119 86 23 111 97
23-16.1	128	1 2 4 8 16 32 64 31 103 43 85 44 82 54 56 88 78 123 125 104 25 112 49
23-16.8	128	1 2 4 8 16 32 64 31 103 43 85 44 86 88 53 38 58 79 83 124 114 123 106
23-16.5532	128	1 2 4 8 16 32 64 31 103 43 49 74 124 7 94 14 50 121 100 88 112 21 61
23-16.172917	128	1 2 4 8 16 32 64 63 71 25 104 30 41 78 112 15 49 119 86 23 111 97 46
24-17.2	128	1 2 4 8 16 32 64 31 103 43 85 44 86 88 53 38 58 79 83 110 124 97 104 114
24-17.4	128	1 2 4 8 16 32 64 31 103 43 85 44 86 88 53 38 58 79 83 124 114 123 106 113
24-17.4552	128	1 2 4 8 16 32 64 31 103 43 49 74 124 7 94 14 50 121 100 88 112 21 61 13
24-17.256531	128	1 2 4 8 16 32 64 63 71 25 104 30 41 78 112 15 49 119 86 23 111 97 46 39

* There are also non-regular resolution V designs, indicated by footnotes in the following tables. These have not been considered for the catalogues.

** The design is called 17-11.6 in the Chen, Sun and Wu catalogue in the paper, but 17-11.38 in the complete enumeration of 64 run resolution IV designs as obtained from the authors (personal communication with D.X.Sun). Numbering in the paper reflects some trade-off choices by the authors regarding MA and MaxC2 criteria, numbering in the complete listing is strictly in terms of MA.

Table 3: Catalogue of smallest MA class 1 clear compromise designs (no entry[i]: resolution V needed)[ii,iii]

Cell entries: design and choice of design columns for G1

m_1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
7 factors	***7-2.1*** ***1 4***	***7-2.1*** ***1 4 5***	***7-2.1*** ***1 4 5 7***						
8 factors	***8-3.1*** ***1 5***	***8-3.1*** ***1 5 8***							
9 factors	9-4.1 1 9	***9-4.2*** ***1 5 9***	***9-3.1*** ***1 4 5 6***	***9-3.1*** ***1 4 5 6 8***	***9-3.1*** ***1 4 5 6 8 9***				
10 factors	10-4.1 1 4	10-4.1 1 4 5	10-4.1 1 4 5 10	***10-4.3*** ***1 5 6 9 10***					
11 factors	11-5.1 1 5	11-5.1 1 5 11	11-5.4 6 8 9 10						
12 factors[iv]	12-6.1 1 5	12-6.2 6 8 9	12-5.1 1 2 3 4	12-5.1 1 2 3 4 5	12-5.1 1 2 3 4 5 6	12-5.1 1 2 3 4 5 6 7	12-5.1 1 2 3 4 5 6 7 10	12-5.1 1 2 3 4 5 6 7 10 11	
13 factors[iv]	13-7.1 4 6	13-7.6 4 10 13	13-6.1 1 3 5 7	13-6.1 1 3 5 7 8	13-6.1 1 3 5 7 8 9	13-6.1 1 3 5 7 8 9 10	13-6.6 1 2 3 5 7 8 9 10		
14 factors[iv]	14-8.1 1 10	14-8.7 5 10 13	14-7.1 1 2 4 5	14-7.1 1 2 4 5 7	14-7.3 1 4 5 8 9 11	14-7.3 4 5 7 8 9 11 12	14-7.94 1 2 3 5 7 8 9 10		
15 factors[iv]	15-9.3 1 10	15-9.9 6 12 15	15-8.1 1 4 5 6	15-8.1 1 4 5 6 11	15-8.34 4 5 7 9 10 13	15-8.1221 1 2 3 5 7 8 9	15-8.1221 1 2 3 5 7 8 9 10		
16 factors	16-10.2 1 10	16-10.8 6 13 16	16-9.1 2 3 5 8	16-9.2 1 4 5 6 11	16-9.1261 4 5 7 8 9 11				
17 factors	17-11.2 1 10	17-11.7 6 14 17	17-10.1 1 4 5 8	17-10.1 1 4 5 8 9	17-10.5846 4 5 7 8 9 11				
18 factors[v]	18-11.1 1 4	18-11.1 1 4 5	18-11.23 2 5 9 10	18-11.95 1 4 5 8 9	?	?	?	?	?
19 factors[v]	19-12.1 1 4	19-12.2 1 8 9	19-12.488 1 8 9 12	?	?	?	?	?	?
20 factors	20-13.1 1 4	20-13.2 4 5 15	?	?	?	?	?	?	?
21 factors	21-14.1 1 2	21-14.4 2 6 17	?	?	?	?	?	?	?
22 factors	22-15.1 1 12	22-15.7 2 6 17	?	?	?	?	?	?	?
23 factors	23-16.1 1 2	23-16.8 2 5 9	?	?	?	?	?	?	?
24 factors	24-17.2 1 12	24-17.4 2 5 9	?	?	?	?	?	?	?

Table 4: Catalogue of smallest MA class 3 clear compromise designs (no entry[i]: resolution V needed)[iii,vi]

Cell entries: design and choice of design columns for G1

m_1	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
7 factors	***7-2.1*** ***4***	***7-2.1*** ***4 5***	***7-2.1*** ***4 5 7***					
8 factors	8-3.1 5	***8-3.1*** ***5 8***						
9 factors	9-4.1 9	***9-4.2*** ***5 9***	***9-3.1*** ***4 5 6***	***9-3.1*** ***4 5 6 8***	***9-3.1*** ***4 5 6 8 9***			
10 factors	10-4.1 4	10-4.1 4 10	***10-4.3*** ***5 6 9***	***10-4.3*** ***5 6 9 10***				
11 factors	11-5.1 11	***11-5.6*** ***6 10***	***11-5.6*** ***6 10 11***					
12 factors[iv]	12-6.4 11	12-6.23 6 12	12-5.1 2 3 4	12-5.1 2 3 4 5	12-5.1 2 3 4 5 6	12-5.1 2 3 4 5 6 7	12-5.1 2 3 4 5 6 7 10	12-5.1 2 3 4 5 6 7 10 11
13 factors[iv]	13-7.3 10	13-7.34 6 13	13-6.1 5 7 8	13-6.1 5 7 8 9	13-6.1 5 7 8 9 10	13-6.6 1 2 5 7 8 9	13-6.6 1 2 5 7 8 9 10	
14 factors[iv]	14-8.4 10	14-8.40[vii] 6 14	14-7.5 7 9 10	14-7.71 6 7 10 11	14-7.94 1 2 5 7 8	14-7.94 1 2 5 7 8 9	14-7.94 1 2 5 7 8 9 10	
15 factors[iv]	15-9.3 10	***15-9.40***[viii] ***6 15***	15-8.150 1 8 9	15-8.423 6 7 10 11	15-8.1221 1 2 5 7 8	15-8.1221 1 2 5 7 8 9	15-8.1221 1 2 5 7 8 9 10	
16 factors	16-10.2 10	***16-10.45***[ix] ***6 16***	16-9.890 1 8 9	16-9.2913 6 7 10 11	16-9.5539 6 7 9 10 11			
17 factors	17-11.2 10	***17-11.38***[x] ***6 17***	17-10.2407 1 8 9	17-10.9040 6 7 10 11	17-10.12633 6 7 9 10 11			
18 factors[v]	18-11.1 4	18-11.1 4 5	18-11.6381 1 8 9	18-11.18050 6 7 10 11	?	?	?	?
19 factors[v]	19-12.10 1	19-12.9648 1 8	19-12.9648 1 8 9	?	?	?	?	?
20 factors	20-13.11 1	20-13.43452 9 10	?	?	?	?	?	?
21 factors	21-14.8 1	21-14.68031 9 10	?	?	?	?	?	?
22 factors	22-15.8509 10	22-15.118181 2 3	?	?	?	?	?	?
23 factors	23-16.5532 10	23-16.172917 2 3	?	?	?	?	?	?
24 factors	24-17.4552 10	24-17.256531 2 3	?	?	?	?	?	?

Table 5: Catalogue of smallest MA class 4 clear compromise designs (entry V[i]: resolution V needed)[iii,xi]

Cell entries: design and choice of design columns for G1

W.l.o.g., G1 is assumed to be the smaller of the two sets G1 and G2. For larger G1, switch roles of G1 and G2.

m_1	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
7 factors	***7-2.1*** ***4***	***7-2.1*** ***4 5***	***7-2.1*** ***4 5 7***							
8 factors	***8-3.1*** ***5***	***8-3.1*** ***5 8***	V	V						
9 factors	9-4.1 9	***9-4.2[xii]*** ***5 9***	***9-3.1*** ***4 5 6***	***9-3.1*** ***1 2 3 7***						
10 factors	10-4.1 4	10-4.1 4 10	***10-4.3*** ***5 6 9***	10-4.1 1 2 3 7	10-4.1 1 2 3 7 10					
11 factors	11-5.1 11	***11-5.6*** ***6 10***	***11-5.6*** ***6 10 11***	***11-5.1*** ***5 6 9 10***	11-5.1 5 6 9 10 11					
12 factors[iv]	12-6.4[xiii] 11	12-6.23[xiii] 6 12	12-5.1 2 3 4	12-5.1 1 8 9 12	12-5.1 1 3 8 9 12	***12-6.1[xiii]*** ***1 2 3 4 7 8***				
13 factors[iv]	13-7.3[xiv] 10	13-7.34[xiv] 6 13	13-6.1 5 7 8	13-6.1 1 2 11 13	13-6.1 1 2 5 11 13	13-6.1 1 2 5 7 11 13				
14 factors[iv]	14-8.4[vii] 10	14-8.40[vii] 6 14	14-7.5 7 9 10	14-7.1 1 3 10 12	14-7.1 1 3 4 10 12	14-7.1 1 3 4 5 10 12	14-7.14 1 3 5 6 8 11 13			
15 factors[iv]	15-9.3[viii] 10	15-9.40[viii] 6 15	15-8.150 1 8 9	15-8.3 7 9 11 14	15-8.10 1 2 10 11 13	15-8.78 1 5 6 8 11 14	15-8.78 1 4 5 6 8 11 14			
16 factors	16-10.2[ix] 10	16-10.45[ix] 6 16	16-9.890 1 8 9	16-9.80 1 2 11 13	16-9.80 1 2 10 11 13	16-9.1413 1 2 3 10 11 12	V	V		
17 factors	17-11.2[x] 10	17-11.38[x] 6 17	17-10.2407 1 8 9	17-10.1036 1 2 11 13	17-10.1036 1 2 10 11 13	17-10.5924 1 2 3 10 11 12	V	V		
18 factors[v]	18-11.1 4	18-11.1 4 5	18-11.6381 1 8 9	18-11.5146 1 2 11 13	18-11.5146 1 2 10 11 13	18-11.14398 1 2 3 10 11 12	?	?	?	
19 factors[v]	19-12.10 1	19-12.9648 1 8	19-12.9648 1 8 9	19-12.11319 2 3 10 12	19-12.12482 1 2 10 11 13	19-12.26381 1 2 3 10 11 13	?	?	?	
20 factors	20-13.11 1	20-13.43452 9 10	?	?	?	20-13.47458 1 2 3 10 11 13	?	?	?	?
21 factors	21-14.8 1	21-14.68031 9 10	?	?	?	?	?	?	?	?
22 factors	22-15.8509 10	22-15.118181 2 3	?	?	?	?	?	?	?	?
23 factors	23-16.5532 10	23-16.172917 2 3	?	?	?	?	?	?	?	?
24 factors	24-17.4552 10	24-17.256531 2 3	?	?	?	?	?	?	?	?

[i] Designs with "?" entries require at least 256 runs; the actual run size is unknown (because a graph-enhanced complete catalogue of resolution IV 256 run designs is not available).

[ii] The actual run size is larger than the Ke et al. (2005) lower bound for the following combinations (numbers of factors with G1 sizes in parentheses):
10 and 11(1), 12(3 to 5), 13(3 to 4), 14(3), 18 to 22(1), 18 (6 to 8), 19 (5 to 7), 20 and 21(4 to 6), 22 and 23 (4 to 5), 24(4). Whenever the lower bound is 256 and a resolution V design is not possible, the actual run size is not known.

[iii] Designs in bold italics can also be obtained from the Ke et al. (2005) article (up to isomorphism).

[iv] For 12 to 15 factors, there is an irregular resolution V design in 128 runs (cf. e.g. Mee 2009, Section 8.2). This can of course be used as well.

[v] For 18 and 19 factors, there is an irregular resolution V design in 256 runs (cf. e.g. Mee 2009, Section 8.2). This can of course be used as well.

[vi] The actual run size is larger than the Ke et al. (2005) lower bound for the following combinations (numbers of factors with G1 sizes in parentheses):
8(3), 10 (1 and 5), 11 (1, 4, 5), 12 (3 to 5), 13 (3,4,8), 14 (3 and 8), 15(8), 16 and 17 (6 to 8), 18 to 22 (1), 18 (5 to 7), 19 (4 to 6), 20 and 21 (3 to 5), 22 (3 and 4). Whenever the lower bound is 256 and a resolution V design is not possible, the actual run size is not known.

[vii] 14-7.1 would do it with its columns 4 and 5 for G1.

[viii] 15-8.1 would do it with its columns 4 and 5 for G1.

[ix] 16-9.2 would do it with its columns 4 and 5 for G1.

[x] 17-10.1 would do it with its columns 4 and 5 for G1.

[xi] The actual run size is larger than the Ke et al. (2005) lower bound for the following combinations (numbers of factors with G1 sizes in parentheses):
10,11(1), 12(3 to 5), 13(3,4), 14(3), 16 to 19 (7+), 18 to 22(1), 20 to 22(3 to 5), 23 to 24 (3 to 4). Whenever the lower bound is 256 and a resolution V design is not possible, the actual run size is not known.

[xii] 9-3.1 would do it with its columns 4 and 5 for G1.

[xiii] 12-5.1 would do it with its columns 2, 2 3, or 1 3 4 8 9 12 for G1.

[xiv] 13-6.1 would do it with its columns 5 and 7 for G1.